Name: KEY Algebra 2 Date: _____ **Chapter 8 Practice Test**

Rational Expressions and Equations

1. The price per person of renting a banquet room varies inversely with the number of people renting the room. It costs $20 per person if 27 people rent the room. About how much will it cost per person if 34 people rent the room?

$y = \frac{k}{x} \rightarrow 20 = \frac{k}{27} \rightarrow 540 = k \rightarrow y = \frac{540}{k}$

$y = \frac{540}{34} = \$15.88$

2. **The variables x and y vary inversely. Use the given values to write an equation relating x and y.**

$x = -6, y = 3$ $\quad y = \frac{k}{x} \rightarrow 3 = \frac{k}{-6} \quad k = -18$

$y = \frac{-18}{x}$

3. **Identify the vertical asymptote(s) of the graph of the function.** $f(x) = \frac{x^2+9}{x^2-x-20}$

vertical asymptotes: denominator = 0. $x^2 - x - 20 = 0$

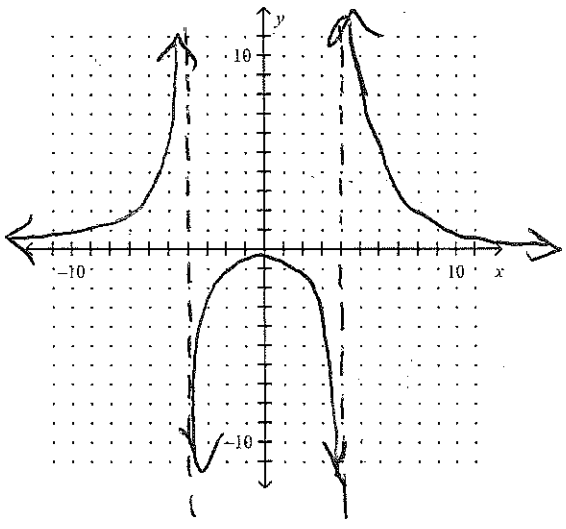
$(x-5)(x+4) = 0$

$x = 5$ and $x = -4$

4. **Simplify the rational expression, if possible.** $\frac{n^2-8n+16}{n^2+2n-24}$

$\frac{(n-4)(n-4)}{(n-4)(n+6)} \rightarrow \frac{(n-4)(n-4)}{(n-4)(n+6)} \rightarrow \frac{n-4}{n+6}$

5. **Graph the function.** $f(x) = \frac{2}{x^2-16}$



x-intercept: none

Vert. Asymptote(s)

$x^2 - 16 = 0$

$(x-4)(x+4) = 0$

$x = 4$ and -4

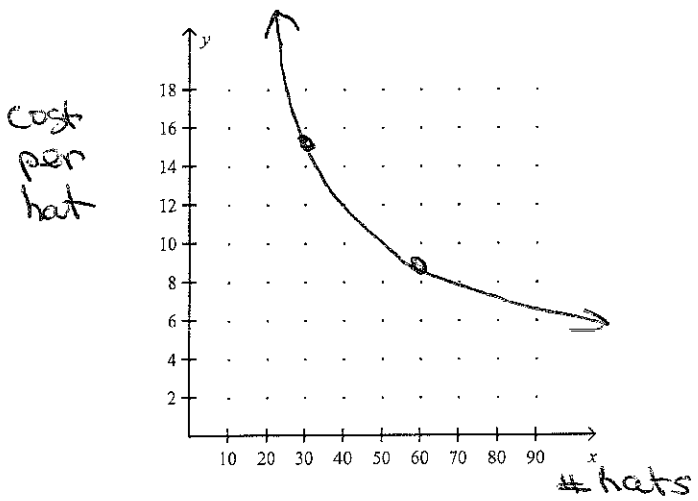
Horizontal Asymptote:

$m = 0$ $\quad m < n$

$n = 2$

$y = 0$ is horizontal asymptote

6. You are selling hats for a fundraiser. The cost of making the designs and buying blank hats is $425. In addition to these one time charges, the cost of printing each hat is $1.75. Let x represent the number of hats that are printed. Write a model that represents the average cost per hat. Then graph the model.



$$y = \frac{425 + 1.75x}{x}$$

Sample points to graph:

$x = 30 \quad y = 15.92$

$x = 60 \quad y = 8.83$

a. State the domain and range: $x > 0$, $y > 0$

b. What is the average cost if 40 hats are produced? $12.38

c. If 80 hats are produced? $7.06

7. **Multiply the expressions. Simplify the result.** $\frac{4y^2}{9} \cdot \frac{18x}{13y} = \frac{4y^{\cancel{2}}}{\cancel{9}_{1}} \cdot \frac{\cancel{18}^{2}x}{\cancel{13y}} = \boxed{\frac{8xy}{13}}$

8. **Multiply the expressions. Simplify the result.** $\frac{(x-1)^2}{x+5} \cdot \frac{x^2+5x}{x^2-1} = \frac{(x-1)(x-1)}{(x+5)} \cdot \frac{x(x+5)}{(x-1)(x+1)}$

$= \frac{\cancel{(x-1)}(x-1)}{\cancel{(x+5)}} \cdot \frac{x\cancel{(x+5)}}{\cancel{(x-1)}(x+1)} = \boxed{\frac{x(x-1)}{(x+1)}}$

9. **Divide the expressions. Simplify the result.** $\frac{3k^2}{9z^3} \div \frac{k^6}{z^7} = \frac{3k^2}{9z^3} \cdot \frac{z^7}{k^6}$

$= \frac{\overset{1}{\cancel{3}}k^{\cancel{2}}}{\underset{3}{\cancel{9}}z^{\cancel{3}}} \cdot \frac{z^{\cancel{7}}z^4}{\underset{k^4}{\cancel{k^6}}} = \boxed{\frac{z^4}{3k^4}}$

10. **Divide the expressions. Simplify the result.** $\dfrac{x^2+10x+21}{x^2-9} \div \dfrac{x+7}{x-7} = \dfrac{(x+7)(x+3)}{(x-3)(x+3)} \cdot \dfrac{(x-7)}{(x+7)}$

$= \dfrac{(x+7)(x+3)}{(x-3)(x+3)} \cdot \dfrac{(x-7)}{(x+7)} = \boxed{\dfrac{(x-7)}{(x-3)}}$

11. **Perform the indicated operation(s) and simplify.** $\dfrac{-3x-3}{45x} + \dfrac{-2x+3}{45x} = \boxed{\dfrac{-3x-3-2x+3}{45x}}$

$= \dfrac{-5x}{45x} = \dfrac{-5x}{9 \cdot 45x} = \boxed{-\dfrac{1}{9}}$

12. **Perform the indicated operation(s) and simplify.** $\dfrac{9}{x+2} + \dfrac{8}{x-2}$ Common Denominator $(x+2)(x-2)$

$\dfrac{9}{x+2} \cdot \dfrac{x-2}{x-2} = \dfrac{9(x-2)}{(x+2)(x-2)}$

$\dfrac{8}{x-2} \cdot \dfrac{(x+2)}{(x+2)} = \dfrac{8(x+2)}{(x+2)(x-2)}$

$\dfrac{9(x-2)+8(x+2)}{(x+2)(x-2)} = \dfrac{9x-18+8x+16}{(x+2)(x-2)}$

$= \boxed{\dfrac{17x-2}{(x+2)(x-2)}}$

13. **Perform the indicated operation(s) and simplify.** $\dfrac{3x}{x^2-16} + \dfrac{3}{x+4} - \dfrac{5}{x-4}$

Common Denominator: $(x+4)(x-4)$

$\dfrac{3}{x+4} \cdot \dfrac{(x-4)}{(x-4)} = \boxed{\dfrac{3(x-4)}{(x+4)(x-4)}}$

$\dfrac{5}{x-4} \cdot \dfrac{(x+4)}{(x+4)} = \boxed{\dfrac{5(x+4)}{(x+4)(x-4)}}$

$\dfrac{3x+3(x-4)-5(x+4)}{(x+4)(x-4)} = \dfrac{3x+3x-12-5x-20}{(x+4)(x-4)}$

$= \boxed{\dfrac{x-32}{(x+4)(x-4)}}$

14. **Solve the equation. Check for extraneous solutions.** $\dfrac{x+2}{2x} - \dfrac{1}{4x} = \dfrac{1}{8}$ $\quad 8x\left[\dfrac{x+2}{2x} - \dfrac{1}{4x} = \dfrac{1}{8}\right]$

$4(x+2) - 2 = x$

$4x+8-2 = x$

$4x+6 = x$

$3x = -6$ $\quad \boxed{x = -2}$

Check: $\dfrac{-2+2}{2(-2)} - \dfrac{1}{4(-2)} = \dfrac{1}{8}$

$0 + \dfrac{1}{8} = \dfrac{1}{8}$ ✓

15. **Solve the equation. Check for extraneous solutions.** $\dfrac{-4}{x+3} = \dfrac{5}{x-3}$

$-4(x-3) = 5(x+3)$

$-4x+12 = 5x+15$

$-3 = 9x \longrightarrow \dfrac{-3}{9} = \boxed{x = -\dfrac{1}{3}}$

Check: $\dfrac{-4}{-\frac{1}{3}+3} = \dfrac{5}{-\frac{1}{3}-3}$

$\dfrac{-4}{2\frac{2}{3}} = \dfrac{5}{-3\frac{1}{3}}$

$-1\frac{1}{2} = -1\frac{1}{2}$ ✓

16. **Solve the equation. Check for extraneous solutions.** $\dfrac{2}{3x} + \dfrac{1}{6} = \dfrac{4}{3x}$

$6x\left[\dfrac{2}{3x} + \dfrac{1}{6} = \dfrac{4}{3x}\right]$

$4 + x = 8$

$\boxed{x = 4}$

Check: $\dfrac{2}{12} + \dfrac{1}{6} = \dfrac{4}{12}$

$\dfrac{1}{6} + \dfrac{1}{6} = \dfrac{1}{3}$

$\dfrac{2}{6} = \dfrac{1}{3}$ ✓

17. **Solve the equation. Check for extraneous solutions.** $\dfrac{k}{k+4}+\dfrac{4}{k-4}=\dfrac{2k+15}{(k+4)(k-4)}$

$(k+4)(k-4)$

$k(k-4)+4(k+4)=2k+15$

$k^2-4k+4k+16=2k+15$

$k^2-2k+1=0$

$(k-1)(k-1)=0$

$\boxed{k=1}$

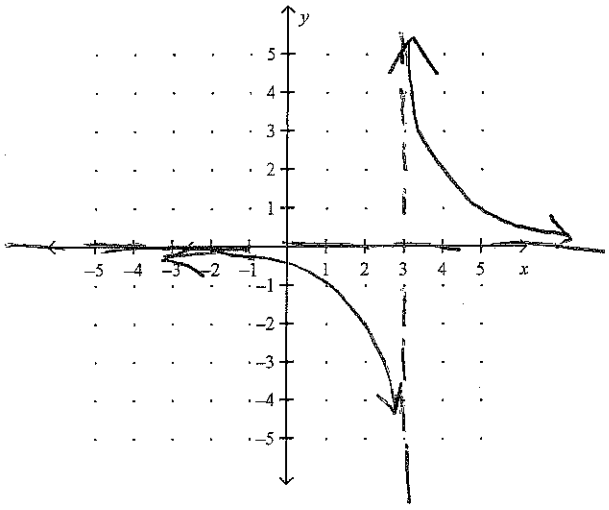
check: $\frac{1}{5}+\frac{4}{-3}=\frac{17}{-15}$

$\frac{3}{15}-\frac{20}{15}=\frac{-17}{15}$ ✓

Sketch the graph of the function. Include any vertical or horizontal asymptotes.

18.

$y=\dfrac{1}{x-3}$



x-int: none

Vert. Asymptote: $x=3$

Horiz. Asymp: $m=0$, $n=1$, $m<n$

$y=0$ is horizontal asymptote